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Compactifications with S-duality Twists

Aybike Çatal-Özer
with Chris Hull

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Wien

Scherk-Schwarz (SS) Reductions are generalisations of Kaluza-Klein (KK) reductions [Scherk, Schwarz, 1979].

Theory must possess a global symmetry, G. The reduction ansatz is determined by the action of the symmetry on the fields.

–Introduces mass parameters

–Introduces a scalar potential

–Supersymmetry Breaking

–Moduli Fixing

–New massive/gauged SUGRA and corresponding string compactifications. (Related with flux compactifications in string theory)

We generalise SS Reductions:

We use S-duality type symmetries for the reduction.

General meaning. A simple example is the electromagnetic duality of the Maxwell equations in the absence of source terms

$$E \to B, \qquad B \to -E$$

Note that it's not a symmetry of the action

$$L \sim E^2 - B^2$$

Interchanges field equations and Bianchi identities

$$dF = 0, \qquad d \star F = 0$$

This is what we mean by S-duality:

– Acts through electromagnetic duality, interchanges field equations and Bianchi identity.

– Is a symmetry of the field equations, but not the action.

EXAMPLES

1) Heterotic String theory on T^6: classical SL(2, R) symmetry:

Montonen-Olive type S-duality symmetry acting on the axion-dilaton fields

$$\tau = \chi + ie^{-\phi} \mapsto \frac{a\tau + b}{c\tau + d}, \quad g_s \mapsto \frac{1}{g_s}$$

strong-weak coupling duality. Symmetry of the field equations only.

2) $D = 11$ SUGRA on T^d: $E_{d,d}$. If $D = 2n$ then a symmetry of the field equations only. e.g.

$$\begin{array}{lcl} D = 4 & \longrightarrow & E_7 \\ D = 6 & \longrightarrow & SO(5,5) \\ D = 8 & \longrightarrow & SL(3,R) \times SL(2,R) \end{array}$$

Why important??

–New gauged SUGRA
–New compactifications:
S-folds (When lifted to string/M/F theory)

Why difficult??

With a symmetry of the field equations, the best we can do is to reduce the field equations... Messy!

STRATEGY

Implement symmetry at the level of an "equivalent" auxiliary Lagrangian and do the SS reduction on the action.

The auxiliary Lagrangian has extra degrees of freedom (dof). The number of dof is kept correct by imposing a self-duality equation.

what do we get?

–Calculations simplified

–Novel features: massive self-duality. CS terms.. Particularly interesting in $D = 4 \to D = 3$ [Nicolai-Samtleben, 2003]

OUTLINE

1. Scherk-Schwarz Mechanism

2. Doubled Formalism

3. Applications

Consider SS on S^1: $D+1 \to D$
(y, x^μ): $S^1 \times M^D$. $y \sim y + 2\pi R$.

$$G : \Phi \mapsto g[\Phi]$$

Scherk-Schwarz ansatz is

$$\Phi(x^\mu, y) = g(y)\Phi(x)$$

$$g : S^1 \longrightarrow G, \qquad g(y) = \exp[M\frac{y}{2\pi R}]$$

MONODROMY:
$\mathcal{M} = \Phi(x, 2\pi R)\Phi^{-1}(x, 0) = e^M \in G$

Classified by conjugacy classes of $\mathcal{M}$.

Mass matrix M determines the mass parameters, the gauge group and the scalar potential of the lower dimensional theory.

$G = SL(2,R)$: 3 conjugacy classes $\Rightarrow$ 3 distinct reductions. The hyperbolic, elliptic and parabolic monodromy and mass matrices:

$$\mathcal{M}_h = \begin{pmatrix} e^m & 0 \\ 0 & e^{-m} \end{pmatrix}, \; \mathcal{M}_e = \begin{pmatrix} \cos m & \sin m \\ -\sin m & \cos m \end{pmatrix},$$

$$\mathcal{M}_p = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}.$$

$$M_h = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}, \quad M_e = \begin{pmatrix} 0 & m \\ -m & 0 \end{pmatrix},$$

$$M_p = \begin{pmatrix} 0 & m \\ 0 & 0 \end{pmatrix}$$

M_e: compact gauging $SO(2)$

M_h, M_p: $SO(1,1)$-gauged lower dimensional theories

Geometrical picture

Φ are sections of $P(M^D \times S^1, G)$. Twist the fibers with $\mathcal{M} \Rightarrow$ non-trivial transition functions.

Useful picture to understand lifting to F/M/string theory. e.g. $G = SL(2, R)$: Compactification on T^2 bundle over S^1 (τ modulus of T^2). –non-trivialisable due to the twist– Interesting when the transition functions are not geometric symmetries. Consistent non-geometric string backgrounds: T-folds, U-folds, S-folds. [Hull, 2004]

Parabolic: T-duality along one of the legs of T^2: untwisted T^3 with H-flux $\longrightarrow$ relation to flux compactifications.

SUGRA, scalars:G/H, H: maximal compact supgroup of G. r $n-1$ form fields A^i_{n-1}, $H^i_n = dA^i_{n-1}$:

$$\mathcal{L} = R * 1 + \frac{1}{4}\mathrm{tr}(d\mathcal{K} \wedge *d\mathcal{K}^{-1}) - \frac{1}{2}H^T_n \mathcal{K} \wedge *H_n$$

$$A \to L^{-1}A, \qquad \mathcal{K} \to L^T \mathcal{K} L$$

Scherk-Schwarz ansatz:

$$\widehat{\mathcal{K}}(x,y) = \lambda^T(y)\mathcal{K}(x)\lambda(y)$$

$$\widehat{A}_{n-1}(x,y) = \lambda^{-1}(y)[A_{n-1}(x) + A_{n-2}(x) \wedge dy].$$

$\lambda(y) = e^{My}$.

Scalar Potential comes from $\partial_y \mathcal{K}^{-1}\partial^y \mathcal{K}$:

$$V(\phi) = -\frac{1}{2}e^{2(D-1)\alpha\varphi}\mathrm{tr}(M^2 + M\mathcal{K}^{-1}M^T\mathcal{K}) * 1$$

Stable only when $\mathcal{M} \in H \longrightarrow$ Minkowski vacuum, no cosm. const., all SUSY is broken. Other $\mathcal{M}$ give BPS domain wall solutions.

$$\widehat{H}_n(x,y) = e^{-My} H_n(x) + e^{-My} H_{n-1}(x) \wedge (dy + \mathcal{A})$$

$$H_{n-1}(x) = dA_{n-2} - (-1)^{n-1} M A_{n-1},$$

$$H_n(x) = dA_{n-1} - H_{n-1} \wedge \mathcal{A}.$$

Kinetic term reduces as:

$$[e^{-2(n-1)\alpha\varphi} H_n^T \mathcal{K} \wedge * H_n + e^{2(D-n)\alpha\varphi} H_{n-1}^T \mathcal{K} \wedge * H_{n-1}]$$

Stückelberg type symmetry:

$$\delta A_{n-1} = d\lambda, \qquad \delta A_{n-2} = (-1)^{n-1} M \lambda$$

Can be used to set $A_{n-2} = 0$ by

$$A_{n-1} \to A_{n-1} + (-1)^{n-1} M^{-1} dA_{n-2}.$$

$$H_n = DA_{n-1} = dA_{n-1} - (-1)^n M A_{n-1} \wedge \mathcal{A}$$

$$H_{n-1} = (-1)^n M A_{n-1} \quad \textit{mass term}$$

$$\delta \mathcal{A} = d\gamma, \quad \delta A_{n-1} = -\gamma M A_{n-1}$$

Gauge group generated by M. Gauge field $\mathcal{A}$

Doubled Formalism: [Cremmer, Julia, Lu, Pope, 1998]

$$\mathcal{L} = -\frac{1}{2} R_{ij} F^i_n \wedge * F^j_n - \frac{1}{2} S_{ij} F^i_n \wedge F^j_n + \mathsf{L}(\Phi)$$

$D = 2n$ k potential fields A^i_{n-1}. $F = dA^i_{n-1}$.
Field Equations: $dG^i = 0$, $G^i = \delta\mathcal{L}/\delta F^i$
Bianchi Identities: $dF^i = 0$

Solve $dG^i = 0$ by introducing k dual potentials $\tilde{A}^i_{n-1}$ (regarded as independent fields), $G = d\tilde{A}^i_{n-1}$
It is possible to construct a manifestly G-invariant Lagrangian $\mathcal{L}(A, \tilde{A})$. $\sharp$ dof is halved by imposing a self-duality constraint

$$H^i_n = J^i{}_j(\phi) * H^j_n \quad G - covariant$$

where

$$J^i{}_j = \Omega^{ik} \mathcal{K}_{kj}. \qquad H_n = \begin{pmatrix} dA_{n-1} \\ d\tilde{A}_{n-1} \end{pmatrix}.$$

Ω is the G invariant matrix. Note $(J*)^2 = 1$.

SS reduction of the constraint (in the fixed gauge):

$$DA_{n-1} = \tilde{M} * A_{n-1}$$

where $\tilde{M} \propto JM$. This is a massive self-duality condition in the odd $D = 2n - 1$ dimensions:

$$\Rightarrow *D * A_{n-1} \propto \tilde{M}^2 * A_{n-1}$$

Can be derived from a Chern-Simons action with mass term:

$$L = P_{ij} A^i \wedge DA^j + \widehat{M}_{ij} A^i \wedge *A^j$$

where $\widehat{M} = P\tilde{M}$, $\quad P_{ij}$:constant matrix.

Number of dof:

$D = 2n$: k massless A^I_{n-1} = 2k massless $A^I_{n-1}, \tilde{A}^I_{n-1}$ + constraint = kc^{2n-2}_{n-1}

$D = 2n - 1$: twisted: k massless A^I_{n-1}+k massless A^I_{n-2} = kc^{2n-2}_{n-1} untwisted: 2k massive A^I_{n-1} + constraint = kc^{2n-2}_{n-1}

$c^s_p = s!/p!(s-p)!$.

D-DIMENSIONAL LAGRANGIAN:

$$\mathcal{L}_D = \mathcal{L}_g + \mathcal{L}_b + \mathcal{L}_s$$

$$\begin{aligned}
\mathcal{L}_g &= R * 1 - \frac{1}{2} d\varphi \wedge * d\varphi - \frac{1}{2} e^{-2(D-1)\alpha\varphi} \mathcal{F}_2 \wedge * \mathcal{F}_2 \\
\mathcal{L}_s &= \frac{1}{4} \mathrm{tr}(\mathcal{D}\mathcal{K} \wedge * \mathcal{D}\mathcal{K}^{-1}) - \\
&\quad - \frac{1}{2} e^{2(D-1)\alpha\varphi} \mathrm{tr}(M^2 + M\mathcal{K}^{-1} M^T \mathcal{K}) * 1 \\
\mathcal{L}_b &= \frac{1}{2} (\Omega^{-1} M)_{ij} [(-1)^{n-1} A_{n-1}^{(i)} \wedge D A_{n-1}^{(j)} + \\
&\quad + e^{2(D-n)\alpha\varphi} \tilde{M}^j{}_k A_{n-1}^{(i)} \wedge * A_{n-1}^{(k)}].
\end{aligned}$$

APPLICATIONS

D=8: Type IIB on T^2 or 11D on T^3.
The classical symmetries are
U-duality: $SL(3,R) \times SL(2,R)_\tau$, τ: complex structure modulus

T-duality: $SL(2,R)_\sigma \times SL(2,R)_\tau$, σ: Kähler modulus, first factor is embedded in $SL(3,R)$.

S-duality: $SL(2,R)_\lambda$, Inherited from S-duality of Type IIB in 10D. Subgroup of $SL(3,R)$ and conjugate to $SL(2,R)_\sigma$.

Scalar fields parametrise:

$$\frac{SL(3,R)}{SO(3)} \times \frac{SL(2,R)}{SO(2)}$$

$(A_3, \tilde{A}_3)$ form an $SL(2,R)$ doublet.

$\star$We use $SL(2,R)_\tau$ to twist the reduction. Symmetry of field equations only and acts on 3-form potential fields through electromagnetic duality transformations.

D=4

D=4 N=8 SUGRA

E_7 duality symmetry of equations of motion. $(A^I, \tilde{A}^I), \quad I = 1, \cdots, 28$ transform under **56** of E_7. 70 scalars: $E_7/SU(8)$.

D=4 N=4 SUGRA coupled to p vector multiplets:

$$O(6, p) \times SL(2, R)$$

$(A^I, \tilde{A}^I)$ $I = 1, \cdots, 6+p$ form 6+p doublets A^{mI} transforming in the (**2, 6+p**). Scalars:

$$SL(2, R)/SO(2) \times O(6, 22)/O(6) \times O(22).$$

$$H^{mI} = Q^m{}_n \mathcal{R}^I{}_J * H^{nJ}$$

$$J^m{}_n = \Omega^{mp} \mathcal{K}_{pn} \text{ and } \mathcal{R}^I{}_J = L^{IK} \mathcal{N}_{KJ}.$$

$$J = Q \otimes \mathcal{R}$$

D=6

D=6 N=8 SUGRA

$SO(5,5)$ duality symmetry of field equations only. 5 -form fields and their duals form a **10** of $SO(5,5)$. The 25 scalars parametrise

$$\frac{SO(5,5)}{SO(5) \times SO(5)}$$

(A truncation) of Type IIB on T^6

$$SL(2,R)_{IIB} \times SL(2,R)_{EM}$$